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18 *Remarks on Auxiliary Tables for Life Contingencies.*

Ages.		103	102	101	100	99
No. living.		3	5	7	9	11
Ages.	Decrements or No. dying.					
104	1	·970874	1·913470	2·828611	3·717098	4·579707
103	2	1·941748	3·826940	5·657222	7·434196
102	2	1·941748	3·826940	5·657222
101	2	1·941748	3·826940
100	2	1·941748
99	2
Sums		·970874	3·855218	8·597299	15·143008	23·439813

The final summations of the series given by the two methods are identical, and the Annuity values for the whole term of life are derived from the division of the sums by the number of living at the given age, as at the head of each column. The above Example will afford an excellent proof of the certainty of the manner in which the laws of probability are applied to the computation of Life Annuities. Let the above numerical results at age of 99 be considered. The Carlisle Table indicates (see 2nd column) that out of 11 persons living at that age, 2 certainly *die before* the end of one year, and a like number out of the original 11, *before* each of the 2nd, 3rd, 4th, and 5th years have respectively elapsed, leaving one survivor at age of 104 who does not outlive one year. An Annuity of £1 is consequently received by each for as many years certain as they survive *complete* years, the discounted value of an Annuity for 1 year certain has then to be multiplied by the number who die in the 2nd year, and the discounted values of Annuities *certain* for 2, 3, 4, and 5 years each, respectively, by the number who die in the 3rd, 4th, 5th, and 6th years. Such present discounted values being summed up, give the worth of 11 Annuities, payable to as many lives of the age of 99; equivalent, as above, to £23·439813; and one-eleventh of this sum, being £2·130892, is the present value of the single life Annuity at that age, according to the above record of observations.

8th Nov., 1850.

F. H.

On the Contrivances required to render Contingent Reversionary Interests Marketable Securities.

WE have been favoured by Mr. Thomson, the Actuary of the Standard Life Assurance Company, and one of the Vice-Presidents of the Institute, with a copy of certain cases submitted to Mr. Edward Sang, and of his opinion in reference thereto. The matters to which they relate are of so much practical importance, and the learning and ability of Mr. Sang are so well known, that we gladly avail ourselves of Mr. Thomson's kind permission to publish

them, and feel we need offer no apology to our readers for giving them entire. We shall have some observations to offer upon each question, and upon the solution of it; but these we defer till the conclusion of the Paper. The first question is as follows:—

B, aged 30, requires an advance of £100, and in consideration thereof offers to convey to the lender a sum of money, payable at the death of A, aged 58, in the event of his surviving A. What sum should be demanded on his death, and B's survivance, so as to allow the lender to secure himself by an assurance of B's life against A's, and to cover interest at the rate of 6 per cent.? Also, what sum should be assured from the commencement, so as to cover the risk entirely throughout, but not to a greater extent than the calculation warrants?

Mr. Sang is requested to give his opinion as much in detail as possible.

OPINION.

The solution of this question must be obtained indirectly by considering the inverse question, "What present advance should be made in consideration of a certain reversionary right to be assigned?" A simple proportion leads from the solution of the one question to that of the other; also, it is most convenient to assume unit as the sum to fall due at the specified succession.

A person, B, possesses the right of succeeding to a fund or property, provided he be alive when a certain event shall happen, and he wishes to dispose of that right or expectation. The person proposing to purchase it, in making his estimate of the price, takes into account the probability of B surviving the specified event, and also demands a rate of interest sufficient to induce him to enter into the transaction, and besides, he weighs the advantage which the seller has over him in an intimate acquaintance with the healths of the parties. Hence, if the reversion be absolute, one rate of interest may be sufficient, while, if the reversion be contingent, a higher rate may be required. In this is the origin of the expedient of converting a contingent into an absolute reversion by means of a Policy of Assurance. The investigation into the health of the borrower, which invariably precedes the granting of a Policy of Assurance, serves, at the same time, to diminish the advantage which he possesses in this respect over the lender. Now, it is clear that since B possesses only the expectation of a contingent reversion, he cannot offer for sale an absolute inheritance, and cannot possibly convey to the purchaser such an expectation unless he be able to induce some third party to undertake the risk of his decease previous to the occurrence of the event on which his expectation depends. Whatever sum he may pay to the Assurer for the underwriting of this risk, it would seem that just by so much will the value of his expectation be increased, and that he will thus have in hand the same balance as if there had been no re-assurance, so that it may be expected to be a matter of indifference to him according to what scheme the Policy of Assurance may be obtained. Such, however, is not the case, for on account of the difference between the rate of interest allowed by the Assurer and that demanded by the lender, the sum paid for the Policy is greater than the allowance made for it, and thus it happens that the re-assurance influences materially the purchase-money of the expectation.

There are, in general, two ways in which the Assurance may be taken, so as to convert the contingent into an absolute reversion, viz., either by assuring the amount as payable at the occurrence of the specified event,

provided B be then dead, or by making it payable at the death of B, should he die before his succession to the fund. The latter is the method proposed in the prefixed memorandum, and is also, as will immediately appear, the preferable one. Yet for the sake of obtaining a general view of the subject, it may be well to consider both cases. These Assurances again may be paid for in a great variety of ways, and it is expedient to contrast the results of several of these modes. For it is reasonable to expect that as the re-assurance is extrinsic to the expectation possessed by B, the manner in which it may be gone about ought not to affect the value of that expectation. The results are, however, discordant, and the *Reporter*, confiding in the perfect consistency of all results obtained by the application of sound principles, cannot avoid the inference that there lurks some imperfection or impropriety in the general features of the transaction.

The present value of the succession of B, aged 30, to A, aged 58, computed at 6 per cent. from the Carlisle Bills, is £40401; but before paying this, the purchaser requires B to effect an Assurance for £1, payable in the event of his death, while A is alive. When B shall have effected this Assurance he will be able to offer for sale an absolute reversion to fall due at the first death of A or B, the value of which, computed also at six per cent., is 50280, and this is the sum which, on the supposition that B has charged himself with the Assurance, the purchaser ought to pay. Now, if the cost of that Assurance, viz., of A's succession to B, be computed also at 6 per cent., it comes out 09879, that is, as is well known, just the difference between the values of the contingent and the absolute reversion. In this way the seller of the reversion receives the same amount whether there be an Assurance or not, and by whatever process the policy may be obtained, or under whatever form it may be taken, provided it result from computations at 6 per cent., the actual balance to be paid to B will still remain 40401. But when the price of the Assurance is computed at a different rate, say 3 per cent., the sum to be received by B varies with the manner in which that Assurance is obtained, as will appear from the examination of the following cases:—

CASE 1.—Let the Assurance for £1, payable at the death of A, provided B be then dead, be purchased by a single payment.

In this case the price of the Assurance computed at 3 per cent. is 09500, while the reversion, as modified by it, is effectively to fall due at the death of A, without any reference to whether B be then dead or alive. Now the value of a reversion absolute at the death of A is 46005; out of this sum, the above price of Assurance has to be defrayed, leaving to B the clear proceeds 36505, so that by this mode of re-assurance, B sustains a loss of 03896 for every £1 of his inheritance, or nearly of 10 per cent. of its actual value.

This loss may also be exhibited thus: the prices of an Assurance of £1 at A's death, if it happen after B's, computed at 3 per cent. and at 6 per cent., are 09500 and 05604. The first is the price at which B is compelled to purchase the Assurance, while the second is that at which he is obliged to sell it—the difference 03896 being the loss which he sustains on this branch of the transaction.

CASE 2.—Let the Assurance for £1, payable at B's death, if it happen before A's, be purchased by a single payment.

The single payment for this Assurance, computed at 3 per cent., is 12730, and the value of the absolute reversion at the first death of A or B,

taken at 6 per cent., is $\cdot50280$, leaving a balance of $\cdot37550$ to B. The loss occasioned by this form of the transaction is thus $\cdot02851$, or about three-fourths of the loss by the former method.

The same result may be obtained by taking the difference between 12730 and $\cdot09879$, the prices computed at 3 and at 6 per cent. of an Assurance of $\pounds 1$ at B's death if before A's.

The difference between the two losses arises in this way; that B in each case is made to borrow a sum of money at 6 per cent., for which he is only allowed 3 per cent. by the Assurance Office. In the first case there is a smaller sum of money borrowed, but that circumstance is more than compensated by the greater length of time over which the loan extends.

CASE 3.—Let the Assurance of $\pounds 1$, payable at B's death, if first, be purchased by uniform annual premiums.

Here we must assume that B does not possess any fund which may furnish security that the premiums shall be regularly paid, and that therefore the purchaser of the reversion must himself advance the premiums as they fall due, expecting 6 per cent. interest on these premiums from the date of their payment.

The annual premium computed at 3 per cent. for an Assurance of $\pounds 1$ at B's death, if first, is $\cdot01144$. The lender, then, is to advance this annually during the joint life of A and B, whence we must multiply it by $9\cdot039$, the value at 6 per cent. of a joint-life annuity, to obtain the present value to the purchaser. This comes out $\cdot10342$, and must be deducted from $\cdot50280$, the value of the reversion of $\pounds 1$ at the first death, leaving to B, as the proceeds of the transaction, $\cdot39938$. The loss in this case is only $\cdot00463$, or hardly one-sixth of that caused by the purchase of the Assurance by a single payment.

The extent and origin of this loss may be better seen by considering, that while $\cdot01144$ is the premium of Assurance computed at 3 per cent., that computed at 6 per cent. is only $\cdot01093$. Now, if the latter premiums had been exacted, there would have been no loss, as the whole transaction would then have been done at 6 per cent.; the difference between the two premiums, which is $\cdot00051$, is thus expended in an annual loss—that is, the lender considers that he must pay $\cdot00051$ annually more than the Assurance is worth to him, and he therefore deducts the present value of that annual payment from the intrinsic value of the reversion. This deduction is $\cdot00051 \times 9\cdot030 = \cdot00461$, as before (the slight difference arises from neglecting the after figures).

In this transaction B borrows annually a sum of money at 6 per cent., to be handed over to the Assurer who allows 3. The whole premium, however, is not borrowed, since the lender receives, and enjoys each year, the additional value of the Assurance. This circumstance, together with the shorter time during which the difference of interest operates, occasions the small comparative loss in this form of the transaction.

Here it is to be noticed that the net 3 per cent. premium has been used, yet the Assurer may charge something additional. The same principles will, however, apply to the actual premium.

(To be continued.)